Exercise 17B: Solutions

1
$$\begin{aligned}\overrightarrow{AB} &= (3\boldsymbol{i} - 5\boldsymbol{j}) - (\boldsymbol{i} + 2\boldsymbol{j}) \\ &= 3\boldsymbol{i} - 5\boldsymbol{i} - \boldsymbol{i} - 2\boldsymbol{j} \\ &= 2\boldsymbol{i} - 7\boldsymbol{j}\end{aligned}$$

2

a
$$\begin{aligned}\overrightarrow{OP} &= \overrightarrow{OA} + \overrightarrow{AP} \\ &= 5\boldsymbol{i} + 6\boldsymbol{j}\end{aligned}$$

b
$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= -5\boldsymbol{i} + 6\boldsymbol{j}\end{aligned}$$

c
$$\begin{aligned}\overrightarrow{BA} &= -\overrightarrow{AB} \\ &= 5\boldsymbol{i} - 6\boldsymbol{j}\end{aligned}$$

3 a $|5\boldsymbol{i}| = \sqrt{5^2} = 5$

b $|-2\boldsymbol{j}| = \sqrt{(-2)^2} = 2$

c
$$\begin{aligned}|3\boldsymbol{i} + 4\boldsymbol{j}| &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9 + 16} = 5\end{aligned}$$

d
$$\begin{aligned}|-5\boldsymbol{i} + 12\boldsymbol{j}| &= \sqrt{(-5)^2 + 12^2} \\ &= \sqrt{25 + 144} = 13\end{aligned}$$

4 a
$$\begin{aligned}\boldsymbol{u} - \boldsymbol{v} &= (7\boldsymbol{i} + 8\boldsymbol{j}) - (2\boldsymbol{i} - 4\boldsymbol{j}) \\ &= 7\boldsymbol{i} + 8\boldsymbol{j} - 2\boldsymbol{i} + 4\boldsymbol{j} \\ &= 5\boldsymbol{i} + 12\boldsymbol{j} \\ |\boldsymbol{u} - \boldsymbol{v}| &= |5\boldsymbol{i} + 12\boldsymbol{j}| \\ &= \sqrt{25 + 144} \\ &= 13\end{aligned}$$

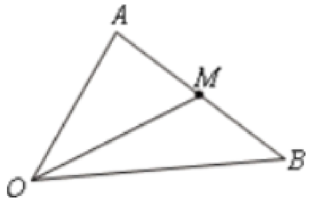
b
$$\begin{aligned}x\boldsymbol{u} + y\boldsymbol{v} &= x(7\boldsymbol{i} + 8\boldsymbol{j}) + y(2\boldsymbol{i} - 4\boldsymbol{j}) \\ &= 7x\boldsymbol{i} + 8x\boldsymbol{j} + 2y\boldsymbol{i} - 4y\boldsymbol{j} \\ &= 44\boldsymbol{j}\end{aligned}$$

$$7x + 2y = 0$$
$$14x + 4y = 0 \qquad (1)$$
$$8x - 4y = 44 \qquad (2)$$

$(1) + (2)$:

$$\begin{aligned}22x &= 44 \\ x &= 2 \\ 7 \times 2 + 2y &= 0 \\ 2y &= -14 \\ y &= -7\end{aligned}$$

5

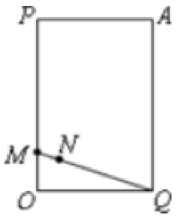


$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB}\\ &= -\overrightarrow{OA} + \overrightarrow{OB}\\ &= -10\boldsymbol{i} + (4\boldsymbol{i} + 5\boldsymbol{j})\\ &= -6\boldsymbol{i} + 6\boldsymbol{j}\end{aligned}$$

$$\begin{aligned}\overrightarrow{AM} &= \frac{1}{2}\overrightarrow{AB}\\ &= -3\boldsymbol{i} + \frac{5}{2}\boldsymbol{j}\end{aligned}$$

$$\begin{aligned}\overrightarrow{OM} &= \overrightarrow{OA} + \overrightarrow{AM}\\ &= 10\boldsymbol{i} + \left(-3\boldsymbol{i} + \frac{5}{2}\boldsymbol{j}\right)\\ &= 7\boldsymbol{i} + \frac{5}{2}\boldsymbol{j}\end{aligned}$$

6



a **i**

$$\begin{aligned}\overrightarrow{OM} &= \frac{1}{5}\overrightarrow{OP}\\ &= \frac{2}{5}\boldsymbol{i}\end{aligned}$$

ii

$$\begin{aligned}\overrightarrow{MQ} &= \overrightarrow{MO} + \overrightarrow{OQ}\\ &= -\overrightarrow{OM} + \overrightarrow{OQ}\\ &= -\frac{2}{5}\boldsymbol{i} + \boldsymbol{j}\end{aligned}$$

iii

$$\begin{aligned}\overrightarrow{MN} &= \frac{1}{6}\overrightarrow{MQ}\\ &= \frac{1}{6}\left(-\frac{2}{5}\boldsymbol{i} + \boldsymbol{j}\right)\\ &= -\frac{1}{15}\boldsymbol{i} + \frac{1}{6}\boldsymbol{j}\end{aligned}$$

iv

$$\begin{aligned}\overrightarrow{ON} &= \overrightarrow{OM} + \overrightarrow{MN}\\ &= \frac{2}{5}\boldsymbol{i} + \left(-\frac{1}{15\boldsymbol{i}} + \frac{1}{6}\boldsymbol{j}\right)\\ &= \frac{1}{3}\boldsymbol{i} + \frac{1}{6}\boldsymbol{j}\end{aligned}$$

v

$$\begin{aligned}\overrightarrow{OA} &= \overrightarrow{OP} + \overrightarrow{PA}\\ &= 2\boldsymbol{i} + \boldsymbol{j}\end{aligned}$$

b i $\vec{ON} = \frac{1}{3}\boldsymbol{i} + \frac{1}{6}\boldsymbol{j}$

$= \frac{1}{6}(2\boldsymbol{i} + \boldsymbol{j})$

$= \frac{1}{6}\vec{OA}$

Since ON is parallel to OA and they share a common point O, ON must be on the line OA. Hence N is on OA.

ii 1:5

7 $\vec{OA} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \boldsymbol{i} + 3\boldsymbol{j}$

$\vec{OB} = \begin{bmatrix} 5 \\ -1 \end{bmatrix} = 5\boldsymbol{i} - \boldsymbol{j}$

$\vec{AB} = -\vec{OA} + \vec{OB}$

$= -\boldsymbol{i} - 3\boldsymbol{j} + 5\boldsymbol{i} - \boldsymbol{j}$

$= 4\boldsymbol{i} - 4\boldsymbol{j}$

$|\vec{AB}| = \sqrt{4^2 + (-4)^2}$

$= \sqrt{16 + 16}$

$= \sqrt{32} = 4\sqrt{2}$ units

8 a $\boldsymbol{i} + 3\boldsymbol{j} = 2\ell\boldsymbol{i} + 2k\boldsymbol{j}$

$2\ell = 1$

$\ell = \frac{1}{2}$

$2k = 3$

$k = \frac{3}{2}$

b $x - 1 = 5$

$x = 6$

$y = x - 4$

$= 2$

c $x + y = 6$ ①

$x - y = 0$ ②

① + ② :

$2x = 6$

$x = 3$

$3 + y = 6$

$y = 3$

d $k = 3 + 2l$

$k = -2 - l$

$3 + 2l = -2 - l$

$3l = -5$

$l = -\frac{5}{3}$

$k = -2 - -\frac{5}{3}$

$= -2 + \frac{5}{3}$

$= -\frac{1}{3}$

9 $\overrightarrow{AB} = \begin{bmatrix} 5-2 \\ 1-3 \end{bmatrix}$

$= \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$= 3\boldsymbol{i} - 2\boldsymbol{j}$

$|\overrightarrow{AB}| = \sqrt{3^2 + (-2)^2}$

$= \sqrt{9+4}$

$= \sqrt{13}$

10a $\overrightarrow{AB} = \boldsymbol{i} + 4\boldsymbol{j} - 3\boldsymbol{i}$

$= -2\boldsymbol{i} + 4\boldsymbol{j}$

b $\overrightarrow{AC} = -3\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{i}$

$= -6\boldsymbol{i} + \boldsymbol{j}$

c $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

$= -6\boldsymbol{i} + \boldsymbol{j} - (-2\boldsymbol{i} + 4\boldsymbol{j})$

$= -4\boldsymbol{i} - 3\boldsymbol{j}$

$|\overrightarrow{BC}| = \sqrt{(-4)^2 + (-3)^2}$

$= \sqrt{16+9}$

$= 5$

11a Let $D = (a, b)$.

$\overrightarrow{AB} = -5\boldsymbol{i} + 3\boldsymbol{j}$

$\overrightarrow{CD} = (a+1)\boldsymbol{i} + b\boldsymbol{j}$

$a + 1 = -5$

$a = -6$

$b = 3$

D is $(-6, 3)$.

b Let $F = (c, d)$.

$\overrightarrow{BC} = -\boldsymbol{i} - 4\boldsymbol{j}$

$\overrightarrow{AF} = (c-5)\boldsymbol{i} + (d-1)\boldsymbol{j}$

$c - 5 = -1$

$c = 4$

$d - 1 = -4$

$d = -3$

F is $(4, -3)$.

c Let $G = (e, f)$.

$\overrightarrow{AB} = -5\boldsymbol{i} + 3\boldsymbol{j}$

$2\overrightarrow{GC} = 2(-1-e)\boldsymbol{i} + 2(-f)\boldsymbol{j}$

$2(-1-e) = -5$

$e = \frac{3}{2}$

$-2f = 3$

$f = -\frac{3}{2}$

G is $\left(\frac{3}{2}, -\frac{3}{2}\right)$.

12 $\overrightarrow{OA} = -\overrightarrow{AO}$

$= -\boldsymbol{i} - 4\boldsymbol{j}$

A is $(-1, -4)$.

B is $(-2, 2)$.

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

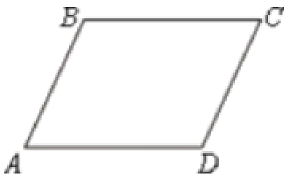
$\overrightarrow{OC} = \overrightarrow{BC} + \overrightarrow{OB}$

$= 2\boldsymbol{i} + 8\boldsymbol{j} + (-2\boldsymbol{i} + 2\boldsymbol{j})$

$= 10\boldsymbol{j}$

C is $(0, 10)$

13



a **i** $2\boldsymbol{i} - \boldsymbol{j}$

ii $-5\boldsymbol{i} + 4\boldsymbol{j}$

iii $\boldsymbol{i} + 7\boldsymbol{j}$

iv $6\boldsymbol{i} + 3\boldsymbol{j}$

v $\overrightarrow{AD} = \overrightarrow{BC}$

$= 6\boldsymbol{i} + 3\boldsymbol{j}$

b $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$\overrightarrow{OD} = \overrightarrow{AD} + \overrightarrow{OA}$

$= 6\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{i} - \boldsymbol{j}$

$= 8\boldsymbol{i} + 2\boldsymbol{j}$

D is $(8, 2)$.

14a $\overrightarrow{OP} = 12\boldsymbol{i} + 5\boldsymbol{j}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= 18\boldsymbol{i} + 13\boldsymbol{j} - 12\boldsymbol{i} - 5\boldsymbol{j}$

$= 6\boldsymbol{i} + 8\boldsymbol{j}$

b $|\overrightarrow{RQ}| = |\overrightarrow{OP}|$

$= \sqrt{12^2 + 5^2}$

$= 13$

$|\overrightarrow{OR}| = |\overrightarrow{PQ}|$

$= \sqrt{6^2 + 8^2}$

$= 10$

15a **i** $|\overrightarrow{AB}| = |2\boldsymbol{i} - 5\boldsymbol{j}|$

$= \sqrt{2^2 + 5^2} = \sqrt{29}$

ii $|\overrightarrow{BC}| = |10\boldsymbol{i} + 4\boldsymbol{j}|$
$= \sqrt{10^2 + 4^2}$
$= \sqrt{116} = 2\sqrt{29}$

iii $|\overrightarrow{CA}| = |12\boldsymbol{i} - \boldsymbol{j}|$
$= \sqrt{12^2 + 1^2} = \sqrt{145}$

b $AB^2 + BC^2 = 29 + 116$
$= 145 = AC^2$

$\therefore$ ABC is a right-angled triangle.

16a i $\overrightarrow{AB} = -\boldsymbol{i} - 3\boldsymbol{j}$

ii $\overrightarrow{BC} = 4\boldsymbol{i} + 2\boldsymbol{j}$

iii $\overrightarrow{CA} = -3\boldsymbol{i} + \boldsymbol{j}$

b i $|\overrightarrow{AB}| = \sqrt{1^2 + 3^2}$
$= \sqrt{10}$

ii $|\overrightarrow{BC}| = \sqrt{4^2 + 2^2}$
$= \sqrt{20} = 2\sqrt{5}$

iii $|\overrightarrow{CA}| = \sqrt{3^2 + 1^2}$
$= \sqrt{10}$

c $AB = CA$
$= \sqrt{10}$
$AB^2 + CA^2 = 10 + 10$
$= 20 = BC^2$

$\therefore$ ABC is an isosceles right-angled triangle.

17a i $\overrightarrow{OA} = -3\boldsymbol{i} + 2\boldsymbol{j}$

ii $\overrightarrow{OB} = 7\boldsymbol{j}$

iii $\overrightarrow{BA} = -3\boldsymbol{i} - 5\boldsymbol{j}$

iv $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BA}$
$= \frac{1}{2}(-3\boldsymbol{i} - 5\boldsymbol{j})$

b $\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM}$
$\overrightarrow{OD} = 7\boldsymbol{j} + -\frac{3}{2}\boldsymbol{i} - \frac{5}{2}\boldsymbol{j}$
$= -\frac{3}{2}\boldsymbol{i} + \frac{9}{2}\boldsymbol{j}$
$M = \left(-\frac{3}{2}, \frac{9}{2}\right)$

18a $\boldsymbol{a} = 3\boldsymbol{i} + 4\boldsymbol{j}$
$|\boldsymbol{a}| = \sqrt{3^2 + 4^2}$
$= 5$

$$\hat{a} = \frac{1}{5}(3\boldsymbol{i} + 4\boldsymbol{j})$$

b

$$\begin{aligned} b &= 3\boldsymbol{i} - \boldsymbol{j} \\ |b| &= \sqrt{3^2 + (-1)^2} \\ &= \sqrt{10} \\ \hat{b} &= \frac{1}{\sqrt{10}}(3\boldsymbol{i} - \boldsymbol{j}) \end{aligned}$$

c

$$\begin{aligned} c &= -\boldsymbol{i} + \boldsymbol{j} \\ |c| &= \sqrt{(-1)^2 + 1^2} \\ &= \sqrt{2} \\ \hat{c} &= \frac{1}{\sqrt{2}}(-\boldsymbol{i} + \boldsymbol{j}) \end{aligned}$$

d

$$\begin{aligned} d &= \boldsymbol{i} - \boldsymbol{j} \\ \hat{d} &= \frac{1}{\sqrt{2}}(\boldsymbol{i} - \boldsymbol{j}) \end{aligned}$$

e

$$\begin{aligned} e &= \frac{1}{2}\boldsymbol{i} + \frac{1}{3}\boldsymbol{j} \\ |e| &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2} \\ &= \sqrt{\frac{1}{4} + \frac{1}{9}} \\ &= \sqrt{\frac{13}{36}} \\ &= \frac{\sqrt{13}}{6} \\ \hat{e} &= \frac{6}{\sqrt{13}}\left(\frac{1}{2}\boldsymbol{i} + \frac{1}{3}\boldsymbol{j}\right) \\ &= \frac{1}{\sqrt{13}}(3\boldsymbol{i} + 2\boldsymbol{j}) \end{aligned}$$

f

$$\begin{aligned} f &= 6\boldsymbol{i} - 4\boldsymbol{j} \\ |f| &= \sqrt{6^2 + (-4)^2} \\ &= \sqrt{52} \\ &= 2\sqrt{13} \\ \hat{f} &= \frac{1}{2\sqrt{13}}(6\boldsymbol{i} - 4\boldsymbol{j}) \\ &= \frac{1}{\sqrt{13}}(3\boldsymbol{i} - 2\boldsymbol{j}) \end{aligned}$$